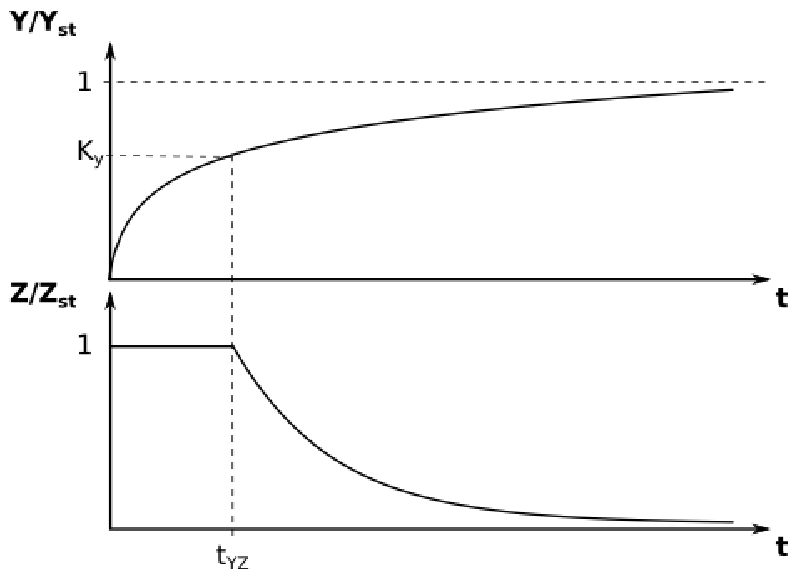
Exercise 06 - Solutions

A cascade of repressors

a) Sketch of the concentrations of Y and Z normalized to their respective max-imum values Y_{st} and Z_{st}.



b) Derivation of $t_{Y_{1/2}}$

$$\begin{aligned}
Y &= Y_{st}(1-e^{-\alpha t}) \\
Y(t=t_{Y_{1/2}}) &= \frac{Y_{st}}{2} \\
\Rightarrow \frac{1}{2} &= 1-e^{-\alpha \cdot t_{Y_{1/2}}} \\
\Rightarrow \frac{1}{2} &= e^{-\alpha \cdot t_{Y_{1/2}}} \\
\Rightarrow ln\left(\frac{1}{2}\right) &= -\alpha \cdot t_{Y_{1/2}} \\
\Rightarrow ln(2) &= \alpha \cdot t_{Y_{1/2}} \\
\Rightarrow t_{Y_{1/2}} &= \frac{ln(2)}{\alpha}
\end{aligned}$$

c) Calculation of t_{YZ}

$$\begin{aligned}
Y(t=t_{YZ}) &= K_y \\
\Rightarrow K_y &= Y_{st}(1-e^{-\alpha \cdot t_{YZ}}) \\
\Rightarrow \frac{K_y}{Y_{st}} &= 1-e^{-\alpha \cdot t_{YZ}} \\
\Rightarrow e^{-\alpha \cdot t_{YZ}} &= 1-\frac{K_y}{Y_{st}} \\
\Rightarrow -\alpha \cdot t_{YZ} &= ln\left(1-\frac{K_Y}{Y_{st}}\right) \\
\Rightarrow \alpha \cdot t_{YZ} &= ln\left(\frac{1}{1-\frac{K_Y}{Y_{st}}}\right) = ln\left(\frac{Y_{st}}{Y_{st}-K_Y}\right) \\
\Rightarrow t_{YZ} &= \frac{1}{\alpha} \cdot ln\left(\frac{Y_{st}}{Y_{st}-K_Y}\right)
\end{aligned}$$

d) Calculation of $t_{Z_{1/2}}$

$$\begin{aligned}
t_{Z_{1/2}} &= t_{YZ} + \frac{ln(2)}{\alpha} \\
&= \frac{1}{\alpha}\left[ln\left(\frac{Y_{st}}{Y_{st}-K_y}\right) + ln(2)\right] \quad ,
\end{aligned}$$

where $\frac{ln(2)}{\alpha}$ is the response time without cascade induced delay.

e) The cascade causes a delay in expression of Z, Y acts as a filter for short pulses of S_X (high-pass filter). In noisy biological systems this can be useful to correctly respond to changes of the input signal X. Stress, changes in the environment, etc., could all cause transient signals S_X. The cells need to be sure that the external change (S_X) is sustained before committing lots of resources to producing Z. Imagine for example Z is the bacterial flagellar motor or the sporulation response.